

# GEOMETRY STANDARDS

The DoDEA high school mathematics program centers around six courses which are grounded by rigorous standards. Two of the courses, AP Calculus and AP Statistics, are defined by a course syllabus that is reviewed and revised on an annual basis. The other 5 courses, Algebra 1, Algebra 2, Geometry, Discrete Mathematics and PreCalculus/Mathematical Analysis, have established standards designed to provide a sequence of offerings that will prepare students for their future goals. These standards serve as the foundation of a comprehensive effort to realize the vision for mathematics education of the students enrolled in DoDEA schools.

**Vision:** DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

## Guiding Principals

### Standards:

- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectation for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

### Instruction:

- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

### Assessment/Accountability:

- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent, and standards-based instructional program that leads to high student achievement.

## Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

<b>Problem Solving</b>	<b>Reasoning and Proof</b>	<b>Communication</b>	<b>Connections</b>	<b>Representation</b>
<p><b>Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:</b></p> <ul style="list-style-type: none"> <li>• build new mathematical knowledge through problem solving;</li> <li>• solve problems that arise in mathematics and in other contexts;</li> <li>• apply and adapt a variety of appropriate strategies to solve problems;</li> <li>• monitor and reflect on the process of mathematical problem solving.</li> </ul>	<p><b>Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:</b></p> <ul style="list-style-type: none"> <li>• recognize reasoning and proof as fundamental aspects of mathematics;</li> <li>• make and investigate mathematical conjectures;</li> <li>• develop and evaluate mathematical arguments and proofs;</li> <li>• select and use various types of reasoning and methods of proof.</li> </ul>	<p><b>Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:</b></p> <ul style="list-style-type: none"> <li>• organize and consolidate their mathematical thinking through communication;</li> <li>• communicate their mathematical thinking coherently and clearly to peers, teachers, and others;</li> <li>• analyze and evaluate the mathematical thinking and strategies of others;</li> <li>• use the language of mathematics to express mathematical ideas precisely.</li> </ul>	<p><b>Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:</b></p> <ul style="list-style-type: none"> <li>• recognize and use connections among mathematical ideas;</li> <li>• understand how mathematical ideas interconnect and build on one another to produce a coherent whole;</li> <li>• recognize and apply mathematics in contexts outside of mathematics.</li> </ul>	<p><b>Instructional programs from Pre-Kindergarten through Grade 12 should enable all students to:</b></p> <ul style="list-style-type: none"> <li>• create and use representations to organize, record, and communicate mathematical ideas;</li> <li>• select, apply, and translate among mathematical representations to solve problems;</li> <li>• use representations to model and interpret physical, social, and mathematical phenomena.</li> </ul>

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## **Strand G.1: Points, Lines, Angles, and Planes**

Students understand the relationship between geometric ideas and their representation.

**Standard:** Students in Geometry will:

**G.1.1:** Demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning;

**Example:** Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates (axioms) as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each.

**Example:** Give an example of how undefined terms are used in the definition of an angle and how the definition of an angle is used in a theorem.

**G.1.2:** Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.

**Example:** Construct the perpendicular bisector of a given line segment, justifying each step of the process.

## **Strand: G.2: Triangles**

Students identify and describe various kinds of triangles (right, acute, scalene, isosceles, etc.) They prove that triangles are congruent or similar and use properties of these triangles to solve problems.

**Standard:** Students in the geometry course will:

**G.2.1:** Identify necessary and sufficient conditions for congruence and similarity in triangles, and use these conditions in proofs;

**Example:** When using the ASA Triangle Congruence Postulate, does it matter which two angles are given? Why or Why not?

**Example:** Triangle NOP has sides twice as long as the sides in triangle KLM. Are the two triangles similar? Why or why not?

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**G.2.2:** Use the triangle angle sum theorem and/or the Pythagorean Theorem and its converse, to solve simple triangle problems and justify results;

**Example:** Starting from his house, Jesse drives north 6 miles and then turns east and drives 2 miles. He turns north again and drives 4 miles and then turns east and drives 7 miles. How far is he from his house? (Hint: You may wish to draw a map.)

**G.2.3:** Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent;

**Example:** Use the Pythagorean Theorem to establish that  $\sin^2 \beta + \cos^2 \beta = 1$  for  $\beta$  between  $0^\circ$  and  $90^\circ$

**G.2.4:** Explain and use the triangle inequality theorem to solve problems;

**Example:** Determine whether you can draw a triangle with sides of length 7 cm, 4 cm, and 15 cm and explain your reasoning.

**G.2.5:** Explain and use angle and side relationships in problems with special right triangles, such as  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  triangles and  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$  triangles.

**Example:** If one leg of a right triangle has length 5 and the adjacent angle is  $30^\circ$ , what is the length of the other leg and the hypotenuse?

**Strand: G.3: Polygons and Circles**

Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.) using terms such as regular, convex, and concave.

**Standard:** Students in the geometry course will:

**G.3.1:** Identify and describe characteristics of convex, concave, and regular polygons;

**Example:** Draw a regular hexagon. Explain how you can determine the measure of each of its angles.

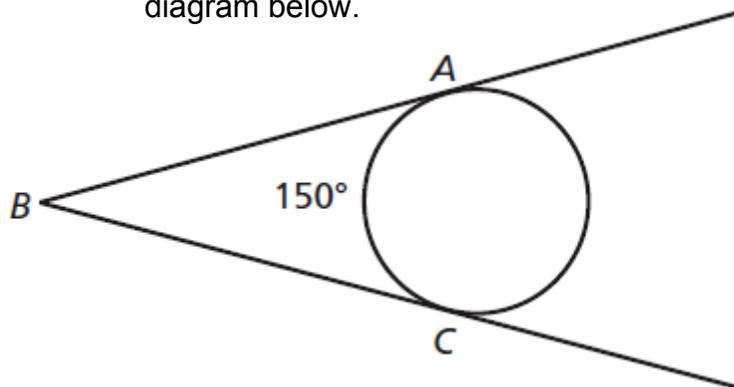
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**G.3.2:** Describe, classify, and explain relationships among the quadrilaterals square, rectangle, rhombus, parallelogram, trapezoid, and kite;

**Example:** Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

**G.3.3:** Identify and determine the measure of central and inscribed angles and their associated minor and major arcs. Recognize and solve problems associated with radii, chords, and arcs within or on the same circle;

**Example:** Find the measure of the angle ABC in the diagram below.



**G.3.4:** Determine the sum of both the interior and exterior angle measures of a polygon;

**Example:** Explain how you can determine the measure of one interior angle of a regular octagon. Use this measure to determine the measure of an exterior angle and calculate the sum of the exterior angles.

**G.3.5:** Determine and use measures of sides and of interior and exterior angles of triangles, quadrilaterals, and other polygons to classify figures, develop mathematical arguments about geometrical relationships, and solve problems.

**Example:** Use a drawing program to create examples of right, acute, obtuse, scalene, isosceles, equilateral, and

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equiangular triangles. Identify and describe the attributes of each triangle.

**Strand: G.4: Coordinate Geometry**

Students understand the relationship of lines and circles with coordinate systems.

**Standard:** Students in the geometry course will:

**G.4.1:** Demonstrate an understanding of the relationship between geometric representation in a coordinate plane and algebraic models of lines and circles;

**Example:** Points A(0,0) and B(3,6) are endpoints of segment AB. Point C(3,1) is an endpoint of segment CD. For point D(1,2), determine whether line AB and line CD are parallel, perpendicular, or neither.

**G.4.2:** Construct the results (using technology when appropriate), and interpret transformations of figures in the coordinate plane, e.g., translations, reflections, rotations, scale factors, and the results of successive transformations. Apply transformations to the solutions of problems;

**Example:** Line  $m$  is described by the equation  $y=2x+3$ . Graph line  $m$  and reflect line  $m$  across the line  $y=x$ . Determine the equation of the image of the reflection. Describe the relationship between the line and its image.

**G.4.3:** Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity;

**Example:** Is the polygon formed by connecting the points (2,1), (6,2), (5,6), and (1, 5) a square?

**G.4.4:** Explain the relationship between scale factors and their inverses and to apply scale factors to scale figures and drawings;

**Example:** The vertices of a figure are (1,3), (2,5), and (4,3) and a scale factor  $n=2$ . Use the dilation  $D(x,y)=(nx,ny)$  to transform the figure and plot the preimage and image on a coordinate plane.

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**G.4.5:** Prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

**Example:** If the endpoints of a segment are  $(-5,2)$  and  $(1, -3)$ , determine the slope and midpoint of the segment. Graph and find the distance between the two points.

**Example:** Given points  $(3,4)$ ,  $(3,-1)$ , and  $(8,-1)$ , find the fourth point to make a rectangle and a fourth point to make a trapezoid.

**Strand: G.5: Measurement**

Students convert between units of measures and use rates and scale factors to solve problems. They compute the perimeter, area, and volume of geometric objects. They investigate how perimeter, area, and volume are affected by changes of scale.

**Standard:** Students in the geometry course will:

**G.5.1:** Determine the perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles;

**Example:** A rectangle of area 360 square yards is ten times as long as it is wide. Find its length and width

**G.5.2:** Given the formula, determine the lateral area, surface area, and volume of prisms, pyramids, spheres, cylinders, and cones;

**Example:** Determine the volume of a sphere with a surface area of 864 square units.

**G.5.3:** Explain how changes in the measurement of one attribute of an object affect changes in other attributes;

**Example:** How does changing the radius from 5 units to 3 units of a cylinder affect its surface area?

**G.5.4:** Determine how changes in dimensions affect the perimeter, area and volume of common geometric figures and solids;

**Example:** What happens to the volume of a rectangular prism if four parallel edges are doubled in length?

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**G.5.5:** Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.

**Example:** The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million ( $3 \times 10^8$ ) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why  $3 \times 10^8$  remained precise to one digit.

**Strand: G.6: Proof and Reasoning**

Students apply geometric skills to making conjectures, using axioms and theorems, understanding the converse and contra positive of a statement, constructing logical arguments, and writing geometric proofs.

**Standard:** Students in the geometry course will:

**G.6.1:** Construct and judge the validity of a logical argument and give counterexamples to disprove a statement;

**Example:** Decide if the following is a plausible conjecture: If segment AM is a median in triangle ABC, then ray AM bisects angle BAC.

**G.6.2:** Prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles;

**Example:** Triangle FGH has vertices at F(1,1), G(2,0), and H(-1,-1). Is triangle FGH a right triangle? Justify your conclusion.

**G.6.3:** Use properties of congruent and similar triangles, quadrilaterals, and other polygons to solve problems;

**Example:** Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

**G.6.4:** Prove and use theorems involving the properties of parallel lines cut by a transversal, similarity, congruence, triangles, quadrilaterals, and circles;

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**Example:** Take two parallel lines  $l$  and  $m$ , with (distinct) points A and B on  $l$  and C and D on  $m$ . If line AC intersects line BD at point E, prove that triangle ABE is similar to triangle CDE.

## Strand G.7: Probability

Students use appropriate statistical methods to analyze data and calculate probability through independent/dependent events and compound events (including geometric probability).

**Standard:** Students in the geometry course will:

**G.7.1** Use sampling or simulation to construct empirical probability distributions to compare and explain corresponding theoretical probabilities;

**Example:** Have students take a strip of paper 10" long and cut it in one place. Determine how many of the cuts resulted in strips that were more than 8" long. Compare this to the theoretical probabilities and generate possible reasons for the difference.

**G.7.2** Describe, create, and analyze a sample space, then calculate the probability;

**Example:** Billy was playing darts on a circular board with a diameter of 60 centimeters. The dartboard is separated into 6 concentric regions by circles whose diameters decrease by 10 centimeters. A dart landing within the inner most circle receives a score of 100. The score for successive region is decreased by 20 points until a dart landing in the outer most region receives no score. At the end of a game with only 1 dart left Billy finds himself behind Joe by 40 and Sally by 80. What is the probability that Billy will finish in 1<sup>st</sup> place? 2<sup>nd</sup> place? 3<sup>rd</sup> place?

**G.7.3** Use the concept of conditional probability and independent events to apply and interpret the results of an event

**Example:** Jose graphs the points (0,0), (8,0), (8,10), and (0,10) and draws in the polygon. Jose then adds the line  $y = x$  to the graph. Complete the graph of the polygon and straight line. What is the probability of picking point inside the rectangle where the x coordinate is larger than the y coordinate?

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**Example:** Have student cut a 10" strip of paper into two parts. Then have the students take the larger of the remaining pieces and cut it again. Determine how many students have a piece that is less than 2" and compare this to the theoretical probabilities.