Algebraic Modeling Standards

The DoDEA high school mathematics program centers on courses which are grounded by rigorous standards. Two of the courses, AP Calculus and AP Statistics, are defined by a course syllabus that is reviewed and revised on an annual basis by the College Board. The other courses, Algebra 1, Algebra 2, Geometry, Discrete Mathematics and Math Analysis, have established standards designed to provide a sequence of offerings that will prepare students for their future goals. New course standards for Algebraic Modeling, Financial Literacy, Engineering Mathematics, and Advanced Functions serve as the foundation of a comprehensive effort to realize the vision for mathematics education of all students enrolled in DoDEA.

Vision: DoDEA students will become mathematically literate world citizens empowered with the necessary skills to prosper in our changing world. DoDEA educators' extensive content knowledge and skillful use of effective instructional practices will create a learning community committed to success for all. Through collaboration, communication, and innovation within a standards-driven, rigorous mathematics curriculum, DoDEA students will reach their maximum potential.

Guiding Principles

Standards:
- Clear and concise standards provide specific content for the design and delivery of instruction.
- Standards provide details that ensure rigor, consistency, and high expectations for all students.
- Standards identify the criteria for the selection of materials/resources and are the basis for summative assessment.

Instruction:
- The curriculum focuses on developing mathematical proficiency for all students.
- The instructional program includes opportunities for students to build mathematical power and balances procedural understanding with conceptual understanding.
- Effective teachers are well-versed in mathematical content knowledge and instructional strategies.
- Classroom environments reflect high expectations for student achievement and actively engage students throughout the learning process.
- Technology is meaningfully integrated throughout instruction and assists students in achieving/exceeding the standards.

Assessment/Accountability
- Assessment practices provide feedback to guide instruction and ascertain the degree to which learning targets are mastered.
- Assessments are used to make instructional decisions in support of the standards and measure standards-based student performance.
- All teachers of mathematics and administrators providing curriculum leadership should be held accountable for a cohesive, consistent and standards-based instructional program that leads to high student achievement.

Mathematics Process Standards

The DoDEA PK-12 mathematics program includes the process standards: problem solving, reasoning and proof, communication, connections, and representation. Instruction in mathematics must focus on process standards in conjunction with all PK-12 content standards throughout the grade levels.

Problem Solving	**Reasoning and Proof**	**Communication**	**Connections**	**Representation**
Instructional programs from pre-kindergarten through grade 12 should enable all students to: • build new mathematical knowledge through problem solving; • solve problems that arise in mathematics and in other contexts; • apply and adapt a variety of appropriate strategies to solve problems; • monitor and reflect on the process of mathematical problem solving.	**Instructional programs from pre-kindergarten through grade 12 should enable all students to:** • recognize reasoning and proof as fundamental aspects of mathematics; • make and investigate mathematical conjectures; • develop and evaluate mathematical arguments and proofs; • select and use various types of reasoning and methods of proof.	**Instructional programs from pre-kindergarten through grade 12 should enable all students to:** • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely.	**Instructional programs from pre-kindergarten through grade 12 should enable all students to:** • recognize and use connections among mathematical ideas; • understand how mathematical ideas interconnect and build on one another to produce a coherent whole; • recognize and apply mathematics in contexts outside of mathematics.	**Instructional programs from pre-kindergarten through grade 12 should enable all students to:** • create and use representations to organize, record, and communicate mathematical ideas; • select, apply, and translate among mathematical representations to solve problems; • use representations to model and interpret physical, social, and mathematical phenomena.

Algebraic Modeling Standards

Strand: AM.1 Linear Equations and Inequalities

Students use technology to model, analyze and apply linear equations and inequalities.

AM.1.1 **Analyze data of real world problems and determine if a linear function appropriately models the data.**

EXAMPLE: Susan is driving from her house to her mother's house in a different town. The trip takes a total of 2 ½ hours. For the first hour, she travels 35 miles through her town. For the second hour, Susan travels 65 miles on the highway. For the last thirty minutes, she travels 20 miles through her mother's town. Should this situation be modeled by a linear function? Justify your answer.

AM.1.2 **Interpret the key characteristics of a linear equation (slope, x- and y-intercepts) in the context of a real world problem.**

EXAMPLE: The student government is having a Valentine's Day dance to raise money. It will cost $500 to rent a local ballroom, and the student government is going to charge $13 per person to attend the dance. Determine the slope and y-intercept, in terms of the profit the committee will make.

AM.1.3 **Model and solve real world problems using a linear equation or inequality; represent the problems both algebraically and graphically.**

EXAMPLE: For Sarah's birthday three months ago, her parents bought her a cell phone. The first month, she talked for 75 minutes and the cost was $16.25. The second month, she talked for 160 minutes and the cost was $30.00. Last month, she talked for 140 minutes, and the cost was $27.00. If Sarah wants to spend less than $22.50 per month, what is the highest number of minutes she can talk? Model the situation both algebraically and graphically.

AM.1.4 **Determine a line of best fit to approximate data that appears to form a linear relationship.**

EXAMPLE: The table below shows the average number of bushels of corn produced per acre (b) and the average number of seeds planted per acre (s) in the past 7 years for a small farm. Model the data both algebraically and graphically. Justify your slope and y-intercept.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Bushels of corn (b)	221	256	215	197	237	223	229
Seed planted (s)	2800	3200	2700	2500	3000	2800	2900

AM.1.5 **Make predictions for real world data based on linear patterns and determine reasonableness.**

EXAMPLE: A marketing company has kept track of the number of boxes of cereal (b) sold per year, in millions, and the amount of money spent on advertising the same cereal (a) each year, in millions. Given the table below, how many boxes of cereal would you predict to be sold if the company spent $9.50 million on advertising? What if the company spent $15.8 million on advertising? Do these answers seem reasonable? Justify both answers.

	Year 1	Year 2	Year 3	Year 4	Year 5
Number of Boxes, in millions (b)	2670	3390	3490	2980	4310
Amount of Adverti sing, in millions (s)	$7.80	$9.90	$10.20	$8.70	$12.60

Strand: AM.2 Systems of Equations and Inequalities

Students use technology to model, analyze and apply systems of equations and inequalities in two variables.

AM.2.1 **Analyze data of real world problems and determine if a system of linear equations or inequalities appropriately models the data.**

EXAMPLE: Two companies have agreed to make shirts for the school's tennis team. Company A will charge a setup fee of $70 and $8.50 per shirt. Company B will charge a $40 set up fee, and $9.25 per shirt. Should this situation be modeled by a system of linear equations? Justify your answer.

Commentary: **Students tend to have difficulty setting up systems of linear equations and seeing that two different situations can be modeled simultaneously. This example illustrates how a student could use a graphing calculator to look at each company and use the model to compare the costs based on the number of shirts.**

AM.2.2 **Interpret key characteristics of linear systems (point of intersection, lines that are parallel, lines that coincide) in the context of a real world problem.**

EXAMPLE: The debate club would like to raise money for their upcoming trip. With permission from the principal, they will sell pizza during lunch. Tony's Pizza will sell them 16 inch pizzas for $7.50. Pizza Roma will sell them three 16 inch pizzas for $22.50. Assuming the pizza quality is the same for each company, which company should the club choose? Model the situation both algebraically and graphically, and justify your answer.

AM.2.3 **Model and solve real world problems using a system of linear equations or inequalities; represent the problems both algebraically and graphically.**

EXAMPLE: Pedro is running for a class office and he would like to print two different types of stickers that say "Vote for Pedro." Each sheet of stickers costs $12, and can make either 6 medium size stickers or 4 large size stickers. If Pedro is willing to spend $100 and wants the same number of medium and large size stickers, what is the maximum number of each he can purchase? Model the situation both algebraically and graphically.

Strand: AM.3 Quadratic Functions

Students use technology to model, analyze and apply quadratic functions.

AM.3.1 **Analyze data of real world problems and determine if a quadratic function appropriately models the data.**

EXAMPLE: From the top of a 48 foot tall building, a ball is thrown straight up with an initial velocity of 32 feet per second. Find the maximum height and the time it takes for the ball to return to the ground. Model the situation both algebraically and graphically.

AM.3.2 **Interpret key characteristics of a quadratic function (domain and range, vertex, maximum/minimum, intercepts, axis of symmetry and end behavior) in the context of a real world problem.**

EXAMPLE: A pipe company produces circular iron disks to be used as endplates for pipes. The cost of the disks is a quadratic function of the diameter. The cost of some disks is given below. Model the situation and determine the key characteristics of the function.

Diameter	1 inch	2 inch	3 inch	4 inch	5 inch
Cost	$12	$18	$28	$42	$60

AM.3.3 **Model and solve real world problems using a quadratic function; represent the problems both algebraically and graphically.**

EXAMPLE: Jay was making a flower garden, and he wanted to enclose it using a decorative fence. He had 16 feet of fencing and wanted to use it all. He decided to make the garden rectangular. Model the situation algebraically and graphically.

AM.3.4 **Determine a curve of best fit to approximate data that appears to form a quadratic relationship.**

EXAMPLE: A college student conducted research about the number of words in a term paper W and number of points P (out of 100) received. Thirty different terms papers, along with the corresponding points awarded, are listed below. Model the data with the curve that best fits the situation. Describe the limitations of your model.

Words	2250	9500	1500	4750	3750	4000	5250	2500	5750	1250	3750	9000	1500	8250	5750
Points	61	34	58	96	93	96	98	81	95	39	90	41	58	62	96

Words	5250	3750	8500	6250	9250	4750	6500	8500	9250	1750	3500	8000	1500	4000	1750
Points	99	89	50	87	41	99	92	63	33	59	90	60	40	95	47

AM.3.5 **Make predictions for real world data based on quadratic relationships and determine reasonableness.**

EXAMPLE: Billboards can be a very effective means of advertising for a restaurant. The effectiveness (E) of the board is determined by the distance D (in miles) the board is from the restaurant. Given the data below, construct a model that will predict the effectiveness of a billboard that is 6.25 miles way and a billboard that is 11.75 miles away. Are your predictions reasonable? Justify your response.

D	6.00	9.50	3.75	6.00	9.50	2.25	8.50	6.75	4.50	9.75	9.25	6.75	5.75	8.00
E	9.5	3.9	9.4	9.6	3.3	7.1	5.5	9.1	9.9	0.8	2.6	9.0	9.8	7.3
D	1.50	1.50	4.25	5.25	1.25	3.00	9.75	4.75	4.00	1.00	7.00	7.25	6.75	9.75
E	5.8	4.2	9.8	10.0	4.0	8.5	1.7	10.0	9.7	3.8	8.8	8.2	8.5	1.7

Strand: AM.4 Exponential, Logarithmic and Power Functions

Students use technology to model, analyze and apply exponential and logarithmic functions.

AM.4.1 **Analyze data of real world problems and determine if an exponential, logarithmic or power function appropriately models the data.**

EXAMPLE: The population in Jacksonville, NC was 80,525 in 2010 and is growing at an annual rate of 2.3%. If this growth rate continues, would it be reasonable to say that in approximately 10 years Jacksonville will reach a population of 100,000? What model (exponential, logarithmic or power) would best justify your answer and explain why?

AM.4.2 **Interpret key characteristics of exponential, logarithmic and power functions (domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease and rate of change).**

EXAMPLE: Assuming the rate of inflation is 3% per year, the predicted price of an item can be modeled by the function $P(t) = P_0(1.03)^t$, where P_0 represents the initial price of the item and *t* is in years. Graph this equation and identify the type of function, domain and range, asymptotes, and rate of change.

AM.4.3 **Model and solve real world problems using an exponential, logarithmic or power function; represent the problems both algebraically and graphically.**

EXAMPLE: Paul purchased a new Ford F-150 for $26,000. The average depreciation of a new vehicle is approximately 15% per year. Write an exponential model that represents the truck's value *y* (in dollars) after *t* years. Graph the model and estimate when the value of the truck will be $19,000.

AM.4.4 **Determine a curve of best fit to approximate data that appears to form an exponential, logarithmic or power relationship.**

EXAMPLE: The data below shows the average height of the American Giant Sunflower *h* (in inches) over time *w* (in weeks). Graph this data and determine the curve that best fits to the approximate data. What type of function does this represent? Predict the height of an American Giant Sunflower in 15 weeks. Use the graph and the data to explain the reasonableness of your prediction.

Weeks of growth (*w*)	1	2	3	4	5	6	7
Height in inches (h)	1.2	3.1	5	14.1	27.5	38	56.4

AM.4.5 **Make predictions for real world data based on an exponential, logarithmic or power relationship and determine reasonableness.**

EXAMPLE: Scientists use the circumference of an animal's femur to estimate the animal's weight. The table shows the femur circumference *C* (in millimeters) and the weight *W* (in pounds) for several animals. Model the data and then predict the weight of an orangutan if the circumference of its femur is 72.6 millimeters. Explain the reasonableness of your prediction.

Animal	**Tiger**	**Panda**	**Kangaroo**	**Rabbit**	**Cat**
***C* (mm)**	119	83	64	20	26
***W* (pounds)**	670	240	119	4.4	9.68

Strand: AM.5 Functions and Relations

Students use technology to model, analyze and apply piecewise functions and their graphs.

AM.5.1 **Analyze a relationship to determine whether or not it meets the criteria of a function. Identify independent and dependent variables, the domain, range and roots.**

EXAMPLE: The Population Growth Rate for Albania is listed below. Graph the data and determine if the data meets the criteria of a function. If it is a function, identify the key characteristics of the function.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Pop Growth Rate	0.26	0.88	1.06	1.03	0.51	0.52	0.52	0.53	0.54	0.55	0.25	0.27

AM.5.2 **Interpret the key characteristics of a piecewise function (intervals of the domain, range, end behavior of each interval) in the context of a real world problem.**

EXAMPLE: A toy company is trying to find the best price for a new toy. The company understands that the higher the price, the fewer number of customers that will buy the toy. The company has determined that profit can be modeled by the following:

$y = \frac{1}{2}x,\ 0 \le x \le 4$ where y is profit in millions
$y = 2x^2 - 30,\ 4 \le x \le 10$ and
$y = .01x + 160,\ x \ge 10$ x is the number of toys sold in millions

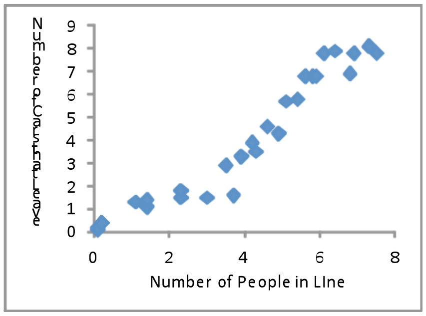
Do these equations represent a piecewise function? Justify your answer.

AM.5.3 **Analyze data of real world problems and determine if a single function or a piece-wise function appropriately models the data.**

EXAMPLE: A telephone company is offering a supersaver policy to provide savings for people who use the telephone very little during the month or a lot during the month. If a person uses the phone for less than or equal to 200 minutes a month, he will pay a flat rate of $30.00. If a person uses the phone more than 200 minutes, he will pay $30.00 plus 40 cents per minute for every minute over 200. If a person uses the phone for more than 400 minutes, he will pay a flat rate of $100.00. Is this situation best modeled with a linear function, quadratic function, exponential function or piecewise function? Justify your answer.

AM.5.4 **Model different situations with a variety of functions (e.g. linear, quadratic, exponential, logarithmic, power, and piecewise) and determine the type of function which best fits the context.**

EXAMPLE: A survey of a local movie theater revealed that the more people that are seen standing in line, the more cars that will enter the parking lot and leave without parking. The graph and data below show the average number of people in line per hour, and the average number of cars that leave per hour. Model this situation with three different functions, and justify which function best fits the situation.



People	0.1	0.1	0.2	1.1	1.4	1.4	2.3	2.3	2.3	3	3.5	3.7	3.9	4.2
Cars	0.1	0.2	0.4	1.3	1.1	1.4	1.8	1.5	1.8	1.5	2.9	1.6	3.3	3.9
People	4.3	4.6	4.9	5.1	5.4	5.6	5.8	5.9	6.1	6.4	6.8	6.9	7.3	7.5
Cars	3.5	4.6	4.3	5.7	5.8	6.8	6.8	6.8	7.8	7.9	6.9	7.8	8.1	7.8

AM.5.5 **Make predictions for a variety of real world data and determine reasonableness.**

EXAMPLE: For the past 12 years, the birth rate in Japan has been decreasing. Some researchers are concerned that such a low birth rate will have a negative impact on the economy. Below is the yearly birthrate for Japan from 2000 to 2011 (number of births per 1000 people). Determine the model that best fits the data and predict when the birth rate will be 5.5 births per 1000 people. Is your prediction reasonable? Given your model, will the birth rate ever reach 0 births per 1000 people?

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Births/1000	9.96	10.04	10.03	9.61	9.56	9.47	9.37	8.10	7.87	7.64	7.41	7.31