Topic Outline for Calculus BC

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)
• An intuitive understanding of the limiting process
• Calculating limits using algebra
• Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior
• Understanding asymptotes in terms of graphical behavior
• Describing asymptotic behavior in terms of limits involving infinity
• Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions
• An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
• Understanding continuity in terms of limits
• Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

* Parametric, polar, and vector functions The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative
• Derivative presented graphically, numerically, and analytically
• Derivative interpreted as an instantaneous rate of change
• Derivative defined as the limit of the difference quotient
• Relationship between differentiability and continuity

Derivative at a point
• Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
• Tangent line to a curve at a point and local linear approximation
• Instantaneous rate of change as the limit of average rate of change
• Approximate rate of change from graphs and tables of values

Derivative as a function
• Corresponding characteristics of graphs of f and f_-
• Relationship between the increasing and decreasing behavior of f and the sign of f_-
• The Mean Value Theorem and its geometric interpretation
• Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives
• Corresponding characteristics of the graphs of f, $f\_$, and $f\_$
• Relationship between the concavity of f and the sign of $f\_$
• Points of inflection as places where concavity changes

Applications of derivatives
• Analysis of curves, including the notions of monotonicity and concavity
+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
• Optimization, both absolute (global) and relative (local) extrema
• Modeling rates of change, including related rates problems
• Use of implicit differentiation to find the derivative of an inverse function
• Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
+ Numerical solution of differential equations using Euler's method
+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives
• Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
• Derivative rules for sums, products, and quotients of functions
• Chain rule and implicit differentiation
+ Derivatives of parametric, polar, and vector functions

III. Integrals

Interpretations and properties of definite integrals
• Definite integral as a limit of Riemann sums
• Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

• Basic properties of definite integrals (examples include additivity and linearity)

* Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen , the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus
• Use of the Fundamental Theorem to evaluate definite integrals
• Use of the Fundamental Theorem to represent a particular antiderivative, and

the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation
• Antiderivatives following directly from derivatives of basic functions
+ Antiderivatives by substitution of variables (including change of limits for
definite integrals), parts, and simple partial fractions (nonrepeating linear
factors only)
+ Improper integrals (as limits of definite integrals)

Applications of antidifferentiation
• Finding specific antiderivatives using initial conditions, including applications to
motion along a line
• Solving separable differential equations and using them in modeling (including
the study of the equation $y=ky$ and exponential growth)
+ Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals Use of Riemann sums (using
left, right, and midpoint evaluation points) and trapezoidal sums to approximate
definite integrals of functions represented algebraically, graphically, and by tables
of values

*IV. Polynomial Approximations and Series

* Concept of series A series is defined as a sequence of partial sums, and
convergence is defined in terms of the limit of the sequence of partial sums.
Technology can be used to explore convergence and divergence.

* Series of constants
+ Motivating examples, including decimal expansion
+ Geometric series with applications
+ The harmonic series
+ Alternating series with error bound
+ Terms of series as areas of rectangles and their relationship to improper
integrals, including the integral test and its use in testing the convergence
of p-series
+ The ratio test for convergence and divergence
+ Comparing series to test for convergence or divergence

* Taylor series
+ Taylor polynomial approximation with graphical demonstration of convergence
(for example, viewing graphs of various Taylor polynomials of the sine function
approximating the sine curve)
+ Maclaurin series and the general Taylor series centered at $x = a$

+ Maclaurin series for the functions $e^x \sin x$, $\cos x$, and $\dfrac{1}{1-x}$

+ Formal manipulation of Taylor series and shortcuts to computing Taylor series,
including substitution, differentiation, antidifferentiation, and the formation of
new series from known series
+ Functions defined by power series
+ Radius and interval of convergence of power series
+ Lagrange error bound for Taylor polynomials