Section P-3: Functions and their Graphs

Domain And Range, Implicit Vs Explicit Functions, Even And Odd Functions

In this section we discuss functions and their properties. Functions play a large part in calculus so understanding the basics now will help you throughout the year. As you probably remember from Algebra II, a **function** is a relation in which each element of the domain is assigned to one and only one element of the range. In other words, each x-value is paired with only one y-value and it will pass the vertical line test when graphed (a vertical line will touch the curve at most at one place). The y is the dependent variable because its value depends on the value of x (the independent variable) which is inserted into the function equation.

Domain and Range

Domain and range are the sets which contain the values which x and y take. Domain contains all numbers which are allowed to be used for x (the independent variable), unless you are told that the domain is further restricted. Range is the set of values which y (the dependent variable) takes when the allowable values of the domain are used. In other words, domain is the set of x-values which can be used and range is the set of values which y actually uses. Remember that range depends on domain. It is important that you get these two sets straight in your brain.

Open VS Closed Intervals

As we talk about domain and range, we often use a number of different notations to denote the required intervals. There are two main types of intervals, open and closed. An **open interval** is one in which the endpoints are not included. This equates to the $>$ or $<$ signs of algebra and the open circles on a number line graph. Parentheses are used to show an open interval. Be careful not to mix it up with an ordered pair! Thus, the open interval (2, 10) is equivalent to the inequality $2 < x < 10$. Although there is nothing wrong with using the inequality signs to denote your interval, the book and the AP exam commonly use the notation with the parentheses, so you need to be familiar with it. A **closed interval** will include the endpoints and the notation uses brackets: [4, 20] includes both endpoints. You can also have half-open intervals where you combine the two symbols, just as you can in algebra. $[2, \infty)$ is equivalent to the algebraic notation $2 \leq x < \infty$ and closed and open dots on a number line. Remember that x can never equal infinity. There is an excellent set of illustrations on page A3 which shows all the different combinations.

Graphing Basic Curves

This section also reviews the shapes of basic curves when graphed. There are some wonderful graphs on page 23 and I have also included all of them in the document after this one: Graphs You Should Know and Recognize. You should be able to graph all of these by plotting points and knowing the basic shapes. You must be able to recognize the shapes of the curves given the functions and vice versa. Know these! However, you also need to know how to use your calculator to graph curves. If you are not comfortable with graphing on the calculator, get out your manual and try some. It is easy and essential for the course.

Now that I have told you that being able to graph on the calculator is essential, I also have to tell you that knowing about the aids to graphing such as symmetry and even/odd functions is also essential. The AP exam will have

sections where you will not be able to use the calculator and will be expected to understand these concepts. You will have to be a well-rounded student, not calculator-shy but not calculator-dependent either.

Transformations

On page 24, the book goes over the basic **transformations** that can be applied to any function. In the AP curriculum you need to be familiar with what happens when you multiply by a constant or add a constant to a function. It is easy enough to figure out, if you just try it; in fact, you should know these from our first exploratory activity too. Look over the list on page 24 and make sure you understand them or go over the document after this one: Graphs You Should Know and Recognize.

Types of Functions

On page 25 we have some basic algebraic terms, that hopefully you already know. I have them summarized here for you:

Elementary Function	There are three types of elementary functions: Algebraic, Trigonometric, and Exponential/Logarithmic.
Polynomial Function	A function in the form: $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where n is the degree of the function, a_i is a coefficient, a_n is the leading coefficient, and a_0 is the constant. This is a type of Algebraic Function.
Rational Function	A function in the form of a fraction. This is a type of Algebraic Function.
Transcendental Function	A function that is not algebraic (i.e. a Trig or Exponential function).

Composite Functions

A **composite function** is formed when you insert one function into another. For any single function, we usually use the notation f (x) = ... or g (x) = ... This notation means that "the y value in this f function will be .... when x has this value". Composite functions give successive instructions. f (g (x)) means first put the given x-value into the g function, and then take that answer and insert it into the f function for x. You always work from the inside out. Alternate notation is sometimes used, so there are three equivalent notations that you need to be familiar with. In each case you start from the inside or right side and work your way out or to the left:

$$f \circ g = (f \circ g)(x) = f(g(x))$$

You have to be careful when naming the domain and range of a composite function. The domain of function f ° g is the set of all x in the domain of g such that g (x) is in the domain of f. In other words, the domain of the composite function will be all of the values of the domain of g that produce g (x) values that also are in the domain of f. You can't let the inside function give you values that cannot be included in the domain of the outside function.

Another caution: the function f (g (x)) is not necessarily equal to the function g (f(x)). In fact they will usually not be equal!

Even and Odd Functions

In section P-1 we talked about symmetry. This will naturally extend to **even and odd functions**. Of the three types of symmetry we looked mainly at two of them, symmetry about the origin and symmetry about the y-axis. The former is a characteristic of an odd function and the latter of an even function. We use the same tests to determine whether or not a function is even or odd that we used in P-1.

If $f(x) = f(-x)$ the function is even and has symmetry about the y-axis.

This means that both x and the opposite of x will yield the same value for y.

If $f(x) = -f(-x)$ the function is odd and has symmetry about the origin.

This means that x and the opposite of x will yield opposite values for y.

Caution!! Do not fall into the trap of thinking that any function with an even exponent will be an even function and that conversely functions with odd exponents will be odd. This is not always true! It is a misconception promoted by some unknown person in order to drive me crazy! You must test the symmetry in order to determine if a function is even or odd. An easy way to do this is to choose two numbers, for instance 2 and -2, and put them into the function. If the answers come out the same, the function is even. If the answers come out opposites, the function is odd. If the answers come out to be neither the same nor opposite, the function is neither even nor odd. For *2 points of extra credit*, send me a Message for Instructor, with the heading Even and Odd, and give me an example of a function with odd exponents that is not an odd function and an example of a function with even exponents that is not even.

Again, I am not going to do any examples for this section. I think you will be able to handle all of the homework problems easily. Let me know if you have any questions. PS - did you find the extra credit I had hidden in these lecture notes?