Similarity

The experimental study of rotations, reflections, and translations in Unit 2 prepares students for the more complex work of understanding the effects of dilations on geometrical figures in their study of similarity in Unit 3. They use similar triangles to solve unknown angle, side length and area problems. Unit 3 concludes with revisiting a proof of the Pythagorean Theorem from the perspective of similar triangles.

Additional Resources

Virtual Manipulatives: http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Protractor and compass available

KHAN ACADEMY https://www.khanacademy.org/

tutor.com http://military.tutor.com/home

Math Terminology

- Angle-Preserving
- Dilation
- Scale Drawing
- Similar
- Similarity Transformation
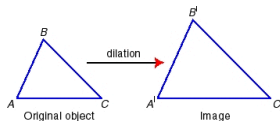
Helpful Tools

Compass

Ruler

Protractor
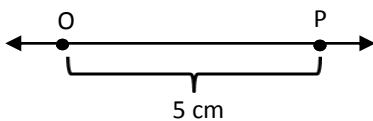
Sample problems are on the back

Big Ideas

- Dilation
- Similar Figures
- The Pythagorean Theorem

New Notations

- The notation |OP| means the length of the segment OP.

  O   P

  5 cm

  |OP| = 5 cm

- The notation Q' mean Q *prime*

  Q ——→ Q'

  For example: point Q after the translation can be denoted as Q'

- Δ ABC ~ Δ DEF means Δ ABC is similar to Δ DEF

College and Career Ready Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

■ **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

■ **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

■ **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
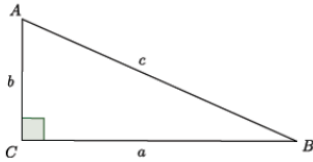
Understand and apply the Pythagorean Theorem.

■ **8.G.6** Explain a proof of the Pythagorean Theorem and its converse.

■ **8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

■ Major Standards | □ Supporting Standards | ○ Additional Standards

Pythagorean Theorem

.

Given the right triangle ABC with C being the vertex of the right angle, then the sides AC and BC are called the legs of the Δ ABC, and AB is called the hypotenuse.

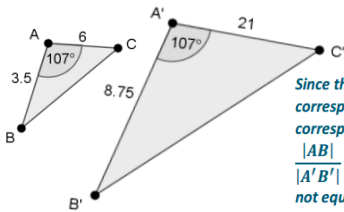$$a^2 + b^2 = c^2$$

Video that demonstrates Pythagorean Theorem proof using similar triangles:

http://www.youtube.com/watch?v=QCyvxYLFSfU

Sample Problem with Similar Triangles

In the diagram below, you have △ ABC and △ A'B'C'. Based on the information given, is △ ABC~ △ A'B'C'?
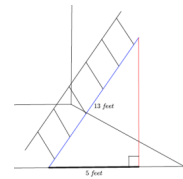
Since there is only information about one pair of corresponding angles, we need to check to see if corresponding sides have equal ratios. That is, does $\frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}$, *or does* $\frac{3.5}{8.75} = \frac{6}{21}$? *The products are not equal;* $73.5 \neq 52.5$. *Since the corresponding sides do not have equal ratios, the triangles are not similar.*

Sample Problem with Pythagorean Theorem

Suppose you have a ladder of length 13 feet. Suppose that to make it sturdy enough to climb, you must place the ladder exactly 5 feet from the wall of a building. You need to post a banner on the building 10 feet above the ground. Is the ladder long enough for you to reach the location you need to post the banner?

Step 1: Draw the picture and label the parts

Step 2: Use Pythagorean Theorem to solve

$$a^2 + b^2 = c^2$$
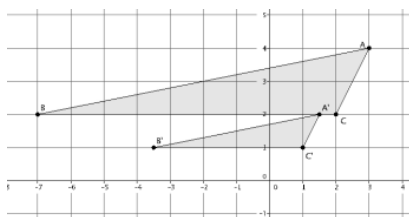$$5^2 + b^2 = 13^2$$
$$b^2 = 13^2 - 5^2$$
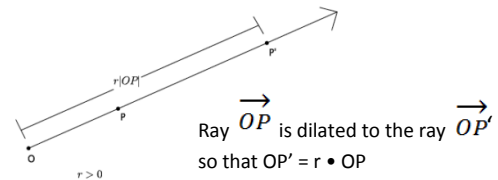$$b^2 = 169—25$$
$$b^2 = 144$$
$$b = 12$$

We now know that the missing side of the triangle is 12 feet. Is the ladder long enough for you to reach the 10-foot banner location? *Yes, the ladder allows us to reach 12 feet up the wall.*

New Math Terminology, Theorems and Examples

- **Dilation**: A dilation is a transformation that produces an image that is the same shape as the original but is a different size. A dilation stretches or shrinks the original figure. The description of a dilation includes the ***scale factor (r)***.

  triangle ABC is dilated from the original triangle A'B'C' by a scale factor of r = 1/2

  Ray $\overrightarrow{OP}$ is dilated to the ray $\overrightarrow{OP'}$ so that OP' = r • OP

- **Similar** Two figures in a plane are similar if there exists a similarity transformation taking one figure onto the other figure. A congruence is a similarity with scale factor 1. It can be shown that a similarity with scale factor 1 is a congruence.

- **AA Criterion for Similarity**: Two triangles with two pairs of equal angles are similar.

- Given just one pair of corresponding angles of a triangle as equal, use the side lengths along the given angle to determine if the triangles are in fact similar.

- Given similar triangles, use the fact that ratios of corresponding sides are equal to find any missing measurements.

  $|\angle A| = |\angle D|$ and $\frac{1}{2} = \frac{3}{6} = r$; therefore, $\triangle ABC \sim \triangle DEF$.

CCRSM Grade 8 Module 3