G8-M1-Lesson 1: Exponential Notation

Use what you know about exponential notation to complete the expressions below.

1. $\underbrace{(-2) \times \cdots \times (-2)}_{35 \text{ times}} = (-2)^{35}$

2. $\underbrace{\left(\frac{9}{2}\right) \times \cdots \times \left(\frac{9}{2}\right)}_{12 \text{ times}} = \left(\frac{9}{2}\right)^{12}$

When the base (the number being repeatedly multiplied) is negative or fractional, I need to use parentheses. If I don't, the number being multiplied won't be clear. Some may think that the 2 or only the numerator of the fraction gets multiplied.

3. $\underbrace{8 \times \cdots \times 8}_{____ \text{ times}} = 8^{56}$

The exponent states how many times the 8 is multiplied. It is multiplied 56 times so that is what is written in the blank.

4. Rewrite each number in exponential notation using 3 as the base.
   a. $9 = 3 \times 3 = 3^2$
   b. $27 = 3 \times 3 \times 3 = 3^3$
   c. $81 = 3 \times 3 \times 3 \times 3 = 3^4$
   d. $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

All I need to do is figure out how many times to multiply 3 in order to get the number I'm looking for in parts (a)–(d).

5. Write an expression with (-2) as its base that will produce a negative product.

One possible solution is shown below.

$(-2)^3 = (-2) \times (-2) \times (-2) = -8$

To produce a negative product, I need to make sure the negative number is multiplied an odd number of times. Since the product of two negative numbers results in a positive product, multiplying one more time will result in a negative product.

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G8-M1-Lesson 2: Multiplication of Numbers in Exponential Form

Let x, a, and b be numbers and $b \neq 0$. Write each expression using the fewest number of bases possible.

1. $(-7)^3 \cdot (-7)^4 =$

   $(-7)^{3+4}$

2. $x^5 \cdot x^6 =$

   x^{5+6}

I have to be sure that the base of each term is the same if I intend to use the identity $x^m \cdot x^n = x^{m+n}$ for Problems 1–4.

3. $\left(\frac{2}{3}\right)^7 \cdot \left(\frac{2}{3}\right)^5 =$

   $\left(\frac{2}{3}\right)^{7+5}$

4. $2^4 \cdot 8^2 =$

   $2^4 \cdot (2^3)^2 =$

   $2^4 \cdot 2^3 \cdot 2^3 =$

   2^{4+3+3}

For this problem, I know that $8 = 2^3$, so I can transform the 8 to have a base of 2.

5. $ab^3 \cdot a^4b^5 =$

   $a \cdot a^4 \cdot b^3 \cdot b^5 =$

   $a^{1+4} \cdot b^{3+5}$

In this problem, I see two bases, a and b. I can use the commutative property to reorder the a's so that they are together and reorder the b's so they are together.

6. $\frac{(-3)^5}{(-3)^2} =$

   $(-3)^{5-2}$

When the bases are the same for division problems, I can use the identity $\frac{x^m}{x^n} = x^{m-n}$.

7. $\frac{a^2b^5}{b^2} =$

   $a^2 \cdot \frac{b^5}{b^2} =$

   a^2b^{5-2}

8. $\frac{27}{3^2} =$

   $\frac{3^3}{3^2} =$

   3^{3-2}

The number 27 is the same as $3 \times 3 \times 3$ or 3^3.

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G8-M1-Lesson 3: Numbers in Exponential Form Raised to a Power

Lesson Notes

Students will be able to rewrite expressions involving powers to powers and products to powers. The following two identities will be used:

For any number x and any positive integers m and n, $(x^m)^n = x^{mn}$.

For any numbers x and y and positive integer n, $(xy)^n = x^n y^n$.

In the lesson today, we learned to use the identity to simplify these expressions. If the directions say show in detail, or prove, I know I need to use the identities and properties I knew before this lesson to show that the identity I learned today actually holds true.

Show (prove) in detail why $(3 \cdot x \cdot y)^5 = 3^5 \cdot x^5 \cdot y^5$.

$(3 \cdot x \cdot y)^5 = (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y)$ ***By the definition of exponential notation***

$= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (x \cdot x \cdot x \cdot x \cdot x) \cdot (y \cdot y \cdot y \cdot y \cdot y)$ ***By the commutative and associative properties***

$= 3^5 \cdot x^5 \cdot y^5$ ***By the definition of exponential notation or by the first law of exponents***

$= 3^5 x^5 y^5$

If I am going to use the first law of exponents to explain this part of my proof, I might want to show another line in my work that looks like this: $3^{1+1+1+1+1} \cdot x^{1+1+1+1+1} \cdot y^{1+1+1+1+1}$.

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G8-M1-Lesson 4: Numbers Raised to the Zeroth Power

Let $x, y, f,$ and g be numbers ($x, y, f, g \neq 0$). Simplify each of the following expressions.

1. $\frac{x^6}{x^6}$

$= x^{6-6}$

$= x^0$

$= 1$

In class, I know we defined a number raised to the zero power as 1.

2. $\frac{x^3y^4}{x^3y^4}$

$= x^{3-3}y^{4-4}$

$= x^0y^0$

$= 1 \cdot 1$

$= 1$

3. $4^5 \cdot \frac{1}{4^5}$

$= \frac{4^5}{4^5}$

$= 4^{5-5}$

$= 4^0$

$= 1$

I have to use the rule for multiplying fractions.

I multiply the numerator times the numerator and the denominator times the denominator.

4. $3^7 \cdot \frac{1}{3^5} \cdot 3^5 \cdot \frac{1}{3^7} \cdot 3^2 \cdot \frac{1}{3^2}$

$= \frac{3^7 \cdot 1 \cdot 3^5 \cdot 1 \cdot 3^2 \cdot 1}{3^5 \cdot 3^7 \cdot 3^2}$

$= \frac{3^{7+5+2}}{3^{5+7+2}}$

$= \frac{3^{14}}{3^{14}}$

$= 3^{14-14}$

$= 3^0$

$= 1$

5. $\frac{f^4 \cdot g^3}{g^3 \cdot f^4}$

$= \frac{f^4 \cdot g^3}{f^4 \cdot g^3}$

$= f^{4-4} \cdot g^{3-3}$

$= f^0g^0$

$= 1 \cdot 1$

$= 1$

6. $(8^2(2^6))^0$

$= (8^2)^0 \cdot (2^6)^0$

$= 8^{2 \cdot 0} \cdot 2^{6 \cdot 0}$

$= 8^0 \cdot 2^0$

$= 1 \cdot 1$

$= 1$

There is a power outside of the grouping symbol. That means I must use the third and second laws of exponents to simplify this expression.

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G8-M1-Lesson 5: Negative Exponents and the Laws of Exponents

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. Compute: $(-2)^4 \cdot (-2)^3 \cdot (-2)^{-2} \cdot (-2)^0 \cdot (-2)^{-2}$

$$= (-2)^{4+3+(-2)+0+(-2)}$$
$$= (-2)^3$$
$$= -8$$

Negative exponents follow the same identities as positive exponents and exponents of zero.

2. Without using (10), show directly that $(y^{-1})^6 = y^{-6}$.

$(y^{-1})^6 = \left(\frac{1}{y^1}\right)^6$ *By definition of negative exponents* (9)

$= \frac{1^6}{y^6}$ By $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (14)

$= \frac{1}{y^6}$

$= y^{-6}$ *By definition of negative exponents* (9)

3. Without using (13), show directly that $\frac{6^{-9}}{6^3} = 6^{-12}$.

$\frac{6^{-9}}{6^3} = 6^{-9} \cdot \frac{1}{6^3}$ *By product formula for complex fractions*

$= \frac{1}{6^9} \cdot \frac{1}{6^3}$ *By definition of negative exponents* (9)

$= \frac{1}{6^9 \cdot 6^3}$ *By product formula for complex fractions*

$= \frac{1}{6^{9+3}}$ By $x^m \cdot x^n = x^{m+n}$ (10)

$= \frac{1}{6^{12}}$

$= 6^{-12}$ *By definition of negative exponents* (9)

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G8-M1-Lesson 6: Proofs of Laws of Exponents

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. A very contagious strain of bacteria was contracted by two people who recently travelled overseas. When the couple returned, they then infected three people. The next week, each of those three people infected three more people. This infection rate continues each week. By the end of 5 weeks, how many people would be infected?

Week of Return	$2+3$
Week 1	$(3\times 3)+(2+3)$
Week 2	$(3^2\times 3)+(3\times 3)+(2+3)$
Week 3	$(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)$
Week 4	$(3^4\times 3)+(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)$
Week 5	$(3^5\times 3)+(3^4\times 3)+(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)$

The 3 people infected upon return each infect 3 people.

Therefore, in week 1, there are 9 new infected people, or $(3\times 3)=3^2$.

Those 9 people infect 3 people each, or 27 new people. $(3^2\times 3)=3^3$

2. Show directly that $r^{-10}\cdot r^{-12}=r^{-22}$.

$r^{-10}\cdot r^{-12}=\frac{1}{r^{10}}\cdot\frac{1}{r^{12}}$ ***By definition of negative exponents*** **(9)**

$=\frac{1}{r^{10}\cdot r^{12}}$ ***By product formula for complex fractions***

$=\frac{1}{r^{10+12}}$ ***By $x^m\cdot x^n=x^{m+n}$ for whole numbers m and n*** **(6)**

$=\frac{1}{r^{22}}$

$=r^{-22}$ ***By definition of negative exponents*** **(9)**

"Show directly" and "prove" mean the same thing: I should use the identities and definitions I know are true for whole numbers to prove the identities are true for integer exponents also.

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