Mathematics Standards for High School —Precalculus

Precalculus is a rigorous fourth-year launch course that prepares students for college and career readiness and intended specifically for those students who plan to pursue a calculus-based course in the future, such as AP Calculus AB/BC in high school, or Business Calculus, calculus-based physics, etc. in postsecondary. Precalculus extends upon learning from Geometry and Algebra II, focusing on trigonometry, polynomial, radical, rational, and exponential functions, and conic sections. There are fewer required standards in the precalculus class, allowing for flexibility in strengthening prerequisite algebraic and numeric understandings and for optional introduction to limits.

Precalculus includes standards from five conceptual categories (Number and Quantity, Algebra, Functions, Modeling, and Geometry). The modeling standards aren't separated into their own category, but dispersed throughout the others and indicated with a ★. Students in this course extend their learning of standards in conceptual categories in Algebra II to include additional (+) standards reserved for fourth-year launch courses such as Precalculus and Discrete Mathematics. There are also three additional standards taken from the California Department of Education, indicated by ending with **CA**.

Precalculus Overview – Conceptual Categories, Domains, and Clusters

Number and Quantity

The Complex Number System

A. Perform arithmetic operations with complex numbers
B. Represent complex numbers and their operations on the complex plane
C. Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

A. Represent and model with vector quantities
B. Perform operations on vectors
C. Perform operations on matrices and use matrices in applications

Algebra

Seeing Structure in Expressions

C. Interpret the structure of expressions

Arithmetic with Polynomials and Rational Expressions

D. Rewrite rational expressions

Creating Equations

B. Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

C. Solve systems of equations

Functions

Interpreting Functions

B. Interpret functions that arise in applications in terms of the context
C. Analyze functions using different representations

Building Functions

B. Build new functions from existing functions

Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle
B. Model periodic phenomena with trigonometric functions
C. Prove and apply trigonometric identities

Geometry

Similarity, Right Triangles, and Trigonometry

D. Apply trigonometry to general triangles

Expressing Geometric Properties with Equations

A. Translate between geometric description and the equation for a conic section

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number and Quantity

The Complex Number System	N—CN
A. Perform arithmetic operations with complex numbers. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **B. Represent complex numbers and their operations on the complex plane.** 4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.* 6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **C. Use complex numbers in polynomial identities and equations.** 8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. 9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	
Vector and Matrix Quantities	**N—VM**
A. Represent and model with vector quantities. 1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, v). 2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. **B. Perform operations on vectors.** 4. (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction ***v*** – ***w*** as ***v + (–w)***, where ***–w*** is the additive inverse of **w**, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 5. (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple c***v*** using ||c***v***|| = |c|v. Compute the direction of c***v*** knowing that when |c|v ≠ 0, the direction of c***v*** is either along ***v*** (for c > 0) or against ***v*** (for c < 0).	

C. Perform operations on matrices and use matrices in applications. 6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 8. (+) Add, subtract, and multiply matrices of appropriate dimensions. 9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	

Algebra

Seeing Structure in Expressions	**A—SSE**
A. Interpret the structure of expressions 1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* 2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*	
Arithmetic with Polynomials and Rational Expressions	**A—APR**
D. Rewrite rational expressions 6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	

Creating Equations★	A—CED
A. Create equations that describe numbers or relationships 1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	
Reasoning with Equations and Inequalities	**A—REI**
C. Solve systems of equations 8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. 9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	

Geometry

Similarity, Right Triangles, and Trigonometry	G—SRT
D. Apply trigonometry to general triangles 9. (+) Derive the formula $A = 1/2\ ab$ sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	
Expressing Geometric Properties with Equations	**G—GPE**
A. Translate between the geometric description and the equation for a conic section 3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	

Functions

Interpreting Functions	F—IF
B. Interpret functions that arise in applications in terms of the context 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 10. (+) Demonstrate an understanding of functions and equations defined parametrically and graph them. **CA** ★ 11. (+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. **CA**	
Building Functions	**F—BF**
B. Build new functions from existing functions 3. Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* 4. Find inverse functions. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	

Trigonometric Functions	F—TF
A. Extend the domain of trigonometric functions using the unit circle 4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **B. Model periodic phenomena with trigonometric functions** 6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. 7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ **C. Prove and apply trigonometric identities** 9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. 10. (+) Prove the half angle and double angle identities for sine and cosine and use them to solve problems. **CA**	